Turing pattern formation on non-normal networks

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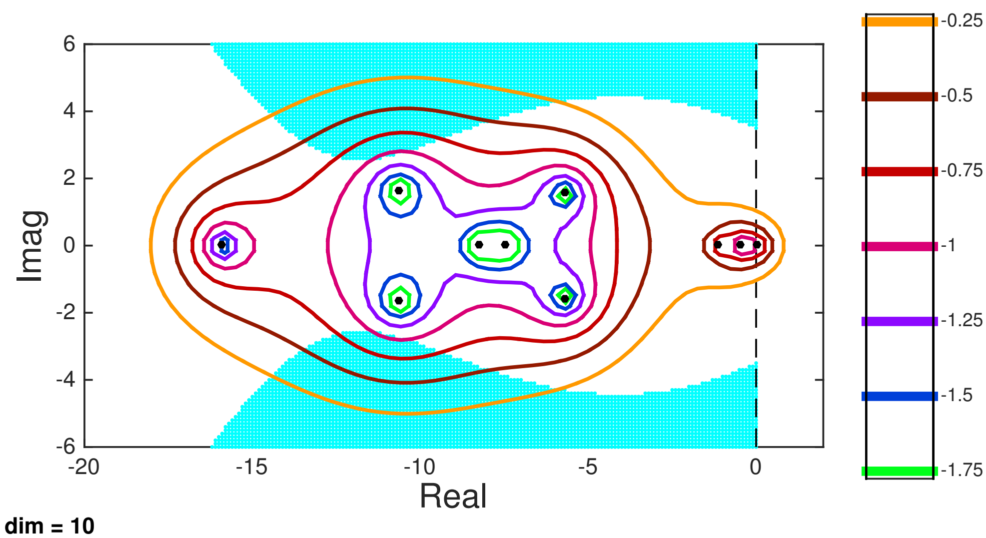
Turing mechanism [1] describes the emergence of spatial patterns in a reaction-diffusion system of two or more species, when perturbations starting from a homogeneous stable state guide the system towards a nonhomogeneous one, the celebrated *Turing patterns*, following a diffusion-driven instability. The classical linear stability analysis that describes such phenomenon is based on the spectra of the involved operators. However, such analysis may fail when the matrices that represent the linear operators are non-normal, i.e. matrices s.t. $AA^* \neq A^*A$. In this case, a new transient growth behavior emerges. In fact, due to the non-normality as measured by the *numerical abscissa*, the system undergoes a transient instability that may generate the global patterns, even though the system itself is stable [2]. This behavior violates the classical Turing understanding of pattern formation. The origin for such discrepancy has been attributed in the past to the strong asymmetric inter-species interactions [3], although such conditions are difficult to be satisfied in realistic scenarios. Starting from these considerations, in this work, we have overcome this difficulty by employing the *non-normal networks* [4], networks which adjacency matrix is a non-normal one. In the light of this new class of networks, allowing the individuals of the species to diffuse on such spatial domain, it is possible to obtain the transient instability out of the Turing parameters region. A technical tool to measure the transient growth induced by a non-normal diffusion operator, is the pseudo-spectra [2], namely the $\epsilon-$perturbated spectra of the Laplacian matrix as shown in figure where the pseudo-spectra curves lie in the instability zone (cyan) although the spectrum (black dots) are outside this region. The method we apply here for pattern formation is more general and can have possible applications in neuroscience, ecology or transport.



References

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