Variable aggregation for dynamical systems on networks

Mauro Faccin, Jean-Charles Delvenne & Michael Schaub

An important problem in complex systems and network science is to find a simplified model of the given complex network in order to gain insight into the high-dimensional topology.
Considering a dynamical system on a network, the dynamical variables are naturally associated with nodes such that we can assign a natural meaning to the dynamical states.
Intuitively in such framework one would like to preserve the inherent characteristics of the nodes. The preferred approach is then to reduce the initial dimensionality through a simple variable aggregation, where nodes get lumped together and form communities or blocks.
In this work I will guide the audience through an information theoretic approach to variable aggregation where the dynamical state-variables are lumped to a lower dimensional space preserving the Markovian fingerprint of the original system while reducing as much as possible its complexity.
In particular this two fold procedure will balance the maximization of the mutual information between two time-steps (non necessarily consecutive) as Markovianity preserving part and the minimization of the variable entropy of the simplified model as the complexity limiting term. The first term is essential in order to avoid the transfer of the topological complexity to the complexity of the dynamics on the simpler model where effective memories could arise due to the variable aggregation.
At this stage we introduce an orthogonal variable aggregation based on the temporal property of the dynamics through a temporal parameter. This acts as a filter for the fast modes of the dynamics usually associated with noise while retaining the important slow modes that characterize the kinetics.
During the talk I will describe how this variable aggregation approach relates to popular community detection algorithms developed from very different approaches (generative models, modularity maximization, eigenvector decomposition, stability).
In the simple case of a cycle of nodes with k-neighbours most of the approaches fail to spot the symmetry of the system and fit the diagonal with maximal cliques, only through the temporal aggregation the right answer can be recovered.